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(for the period April 1, 1975 to May 31, 1979)

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

For the Research Program on

NONPARAMETRIC AND SEQUENTIAL

ANALYSIS OF LIFE TESTING AND

RELIABILITY PROBLEMS

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and Reliability)

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The research project "Non parametric and Sequential Analysis of Life Testing and Reliability Problems" started in April 1, 1975 and was completed in May 31, 1979. During this period the principal investigator, along with his collaborators (who were not supported by AFOSR), considered a number of problems. We describe them below briefly. Lists of publications and technical reports prepared under the project are given in the appendix.

1) Classification and selection rules for the exponential populations

Let $\pi_0, \pi_1, \ldots, \pi_k$ be k+1 different exponential populations. The problem of classification is to classify π_0 as π_i for exactly one i(i-1,2,...,k). Previously we have developed classification procedures for the one parameter exponential distributions. (Classification rules for the exponential populations, by A. P. Basu and A. K. Gupta (1973) published in Reliability and Biometry: Statistical Analysis of life length, ed. Proschan and Serfling, SIAM p. 637-650). We have extended many of the results for one-parameter exponential distributions to the case of the two-parameter exponential distributions [4]. Classification rules based on the likelihood ratio criterion for the two-parameter exponential populations have been studied. These classification rules have been extended to many populations and to the situation where observations are censored. In many cases where the likelihood-ratio criterion does not lead to appealing classification rules, ad hoc rules are proposed. An additional rule is considered from "life testing" point of view. In each case, the probability of misclassification is derived exactly where the parameters are known. The rules considered have been shown to possess a consistency property. An alternative approach based on Bayesian considerations is also explored.

A nonparametric classification problem as described below, has also been considered.

2) Nonparametric analysis of some reliability problems.

Suppose a random sample $X_1, X_2, \ldots, X_m$ of m units with failure time distribution $F(x)$ is put on test. Similarly, let a second sample $Y_1, Y_2, \ldots, Y_n$ of n units from $G(y)$ be tested. To test the hypothesis

$$H_0: \quad F = G$$

against the alternative that they are different, based on the first r ordered observations (failure times) out of a combined sample size m+n, the following statistic is considered.

$$V_r = \sum_{i=1}^{r} (nm_i - mn_i) + \frac{1}{2}(N-r-1)(nm_r - mn_r)\ (r \le N)$$

where m_i and n_i are the number of x and y failures, respectively, among the first i ordered observations of the combined (x,y) sample.

Properties of V_r and its various k-sample extensions are reviewed. Suitable modification of V_r is used to classify an unknown population into one of two given populations. The classification rule is shown to be consistent and its asymptotic relative efficiency with respect to a classification rule for the normal distribution is computed. The problem of estimating the reliability function $P(Y<X)$ is also discussed.

3) Bayesian test for increasing failure rate;

In situations where a statistical analysis is carried out on life-testing data, it is often useful to study the failure rate of the population. The failure rate, $q(t,x)$, of a population over the time interval $(t,t+x)$ is defined as the ratio of the probability that an item selected at random from the population will fail in the time interval $(t,t+x)$, given that it survived up to time t, divided by x. It has been found, both by theoretical and practical investigators, that many of the probability distributions useful in life-testing have either a monotone increasing or monotone decreasing (or constant) failure rate. That is, $q(t,x)$ is a monotone function of t for fixed x. Several tests have been developed by others to test if a population has monotone failure rate. In this article inference for $q(t,x)$ is developed from a Bayesian viewpoint. The failure rate for a future observation is obtained based on an existing data set. Bounds on the posterior probabilities of a distribution with increasing or decreasing failure rate are also obtained. The methods can be used with random samples or multiply censored samples.

4) Estimation of the restricted scale parameter of the exponential distribution.

Let $X_1, X_2, \ldots, X_n$ be failure times of n items put on test and let the density function of the underlying life distribution be given by

$$f(x,\theta) = \frac{1}{\theta} e^{-x/\theta}, \quad x>0 .$$

Suppose, based on past data, it is known that $\theta>\theta_o$, where θ_o is known. Such a situation arises naturally in many physical problems, where it may not be possible to make specific statement about individual observations, but usually it is possible to guarantee a minimum value for the mean life. Thus in a life testing context there might be an early failure due to production error, or due to the presence of some outliers, but overall mean life may be known to be greater than some minimum value.

Several estimators of θ are being considered and their properties are being studied.

5) Nonparametric tests for independence.

The object of this study is to consider rank tests for independence with possible applications in life testing and biomedical problems. The work has been carried out in two different directions.

The first direction is to investigate properties of rank tests under appropriate models. Under alternatives of bivariate normality, performance of such tests, specifically of Kendall's τ, Spearman's ρ, and the normal scores statistic, is well known. In the area of life testing more interest is given to distributions of the bivariate exponential type. Of these, those possessing the lack of memory property are of special interest, for example, the models of Marshall and Olkin (BVE) and Block and Basu (ACBVE).

Testing for independence under the BVE distribution is shown to reduce to that of the ACBVE. For the ACBVE model, Pitman asymptotic relative efficiencies are calculated for Kendall's τ, Spearman's ρ, the maximum likelihood statistic, and the statistic providing the uniformly most powerful test. An appropriate exponential scores statistic and the locally most powerful rank statistic are derived. Approximate large sample power is calculated, and small sample power is investigated using Monte Carlo methods.

Similar but less extensive work is done for a special case of the trivariate extensions of the BVE and ACBVE distributions. Two parametric tests, one the likelihood ratio, are derived as are an exponential scores statistic and the locally most powerful rank statistic. Power studies are carried out for these statistics and for a trivariate generalization of Spearman's ρ.

The general trivariate extensions with the restriction of identical marginals are discussed along with the resulting likelihood ratio test. The results are easily extended to the k-variate situation.

A second direction has been to use suitably modified rank tests to analyze censored data arising from life testing and other biomedical situations. An extensive computer study is done to investigate the behavior of several statistics which are modifications of Kendall's τ adjusted for censoring. Power simulations are done for both the ACBVE and bivariate normal distributions subjected to random censoring in both variables. These statistics are then applied to test for independence in heart pacemaker patient survival data.

6) Estimate of reliability in the stress-strength model.

Suppose Y is the strength of a component which is subject to a stress S. Then the component fails whenever $X \geq Y$, and there is no failure when $X < Y$. The problem of estimating the reliability function

$$R = P(X < Y)$$

is considered. A survey of available results is presented and some new results are considered when X and Y follow gamma, exponential or Weibull distribution. The case when X and Y follow a bivariate exponential distribution is also considered.

7) Estimates of reliability for k-out-of-m systems.

Let S be a system of m independent components $s_1, s_2, \ldots, s_m$ such that the system operates if and only if at least k of these m components successfully operate. Component lives are assumed to follow independent exponential distributions with unknown scale parameters. The minimum variance unbiased estimator of the reliability of the above system at a given mission time t is obtained. The performance of the estimator, when the component lives are identically distributed, is compared with the corresponding maximum likelihood estimator for both large and small samples. Finally, a confidence interval for system reliability is obtained.

8) Identifiability of the multinormal and other distributions under competing risks model.

Let $X_1, X_2, \ldots, X_p$ be p random variables with joint distribution function $F(x_1, \ldots, x_p)$. Let $Z = \min(X_1, X_2, \ldots, X_p)$ and $I = i$ if $Z = X_i$. In this paper the problem of identifying the distribution function $F(x_1, \ldots, x_p)$, given the distribution Z or that of the identified minimum (Z, I), has been considered when F is a multivaraiate normal distribution. For the case p = 2, the problem is completely solved. If p = 3 and the distribution of (Z,I) is given, we get a partial solution allowing us to identify the independent case. These results seem to be highly nontrival and depend upon Liouville's result that the (Univariate) normal distribution function is a nonelementary function. Some other examples are given including the bivariate exponential distribution of Marshall and Olkin, Gumbel, and the absolutely continuous bivariate exponential extension of Block and Basu.

Concluding Remark

The present project solved a number of problems. It also raised new ones. Some of these will be studied further by the principal investigator in the future.

APPENDIX

List of publications and Technical Reports prepared under the project.

1. On characterizing univariate and multivariate exponential distributions with applications (with H. Block). Statistical Distributions in Scientific Work, Vol. 3 (ed. by Patil, et al.), (1975), 399-421.

2. A Bayesian test for increasing failure rate (with R. Lochner). The Theory and Applications of Reliability, Vol. I, ed. by Tsokos and Shimi. Academic Press.

3. A generalized Wilcoxon Mann-Whitney statistic with some applications in reliability. The Theory and Applications of Reliability, Vol. I, (1977), 131-149, ed. by Tsokos and Shimi. Academic Press.

4. Classification rules for exponential populations: two parameter case. The Theory of Applications of Reliability, Vol. I, (1977), 507-525. Ed. by Tsokos and Shimi. Academic Press.

5. Identifiability of the multinormal distribution under competing risk model (with J. K. Ghosh). Jour. of Multivariate Analysis 8, (1978), 413-429.

6. Estimates of reliability of k-out-of-m structures in the independent exponential case (with A. H. ElMaanziny). Jour. Amer. Stat. Association 73, (1978), 850-854.

7. Estimation of restricted scale parameter of the exponential distribution, (with A. K. Gupta), Technical Report No. 49, (1977), University of Missouri-Columbia.

8. On tests of independence under bivariate exponential models (with D. R. Weier). Tech. Report No. 76, (1978), University of Missouri-Columbia.

9. Testing for independence in multivariate exponential distributions (with D. R. Weier). Tech Report No. 77, (1978), University of Missouri-Columbia.

10. A trivariate generalization of Spearman's ρ (with D. R. Weier). Tech. Report No. 78, (1978), University of Missouri-Columbia.

11. An investigation of Kendall's τ modified for censored data with applications (with D. R. Weier). Tech Report No. 79, (1978), University of Missouri-Columbia.